Stock-Flow-Consistent Modeling: Lecture 3 – A simple SFC model

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Simple economy with government money

This example is taken from Godley-Lavoie (2007).

Consider the simplest possible closed economy without a proper banking sector and without investment. Output is given by a homogeneous good demanded by the private sector and the government.

We thus have three sectors: Households; Firms; and the Government.

The government issues a legal currency, which is the only financial asset.

Model matrices

	Hous.	Firms	Gov.	Total
Money	+H		-H	0

	Hous.	Firms	Gov.	Σ
Consumption	-C	+C		0
Gov. expenditure		+G	-G	0
[Output]		[Y]		
Wages	+WB	-WB		0
Taxes	-T		+T	0
Change in fin.assets	$+\Delta H$		$-\Delta H$	0
Σ	0	0	0	0

A "behavioral" matrix

	Hous.	Firms	Gov.	Σ
Consumption	-Cd	+Cs		0
Gov. expenditure		+Gs	-Gd	0
[Output]		[Y]		
Wages	+W.Ns	-W.Nd		0
Taxes	-Ts		+Td	0
Change in fin.assets	+ΔHd		-ΔHs	0
Σ	0	0	0	0

Model identities and equations

$$Y = C + G = WB$$

$$Y = WB \ll WB = W.Ns = Y$$

$$Cs = Cd$$

$$Gs = Gd$$

$$Ts = Td$$

$$Ns = Nd$$

$$YD = W.Ns - Ts$$

Demand-led model

Further assumptions

$Td = \theta.W.Ns \quad \{0 < \theta < 1\}$

$Cd = \alpha 1.YD + \alpha 2.H_{-1} \quad \{0 < \alpha 2 < \alpha 1 < 1\}$

$\Delta Hs = Hs - Hs_{-1} = Gd - Td$

$\Delta Hd = Hd - Hd_{-1} = YD - Cd$

$Y = Cs + Gs$

$Y = W.Nd \quad >> Nd = Y/W$

Model variables

Endogenous

Cd, Cs, Gs, Hd, Hs, Nd, Ns, Td, Ts, Y, YD

Exogenous

Gd, W

Parameters

$\alpha1$, $\alpha2$, θ

The "missing equation"

Stock-flow-consistency implies identities from the matrices. For a SAM with K rows and K columns there will be K*3 identities:

Ri = sum of elements on i-th row

Ci = sum of elements on i-th column

Ri = Ci

However, one of the identities can be derived from the others, so one identity in the system must be dropped

The "missing equation" in model SIM

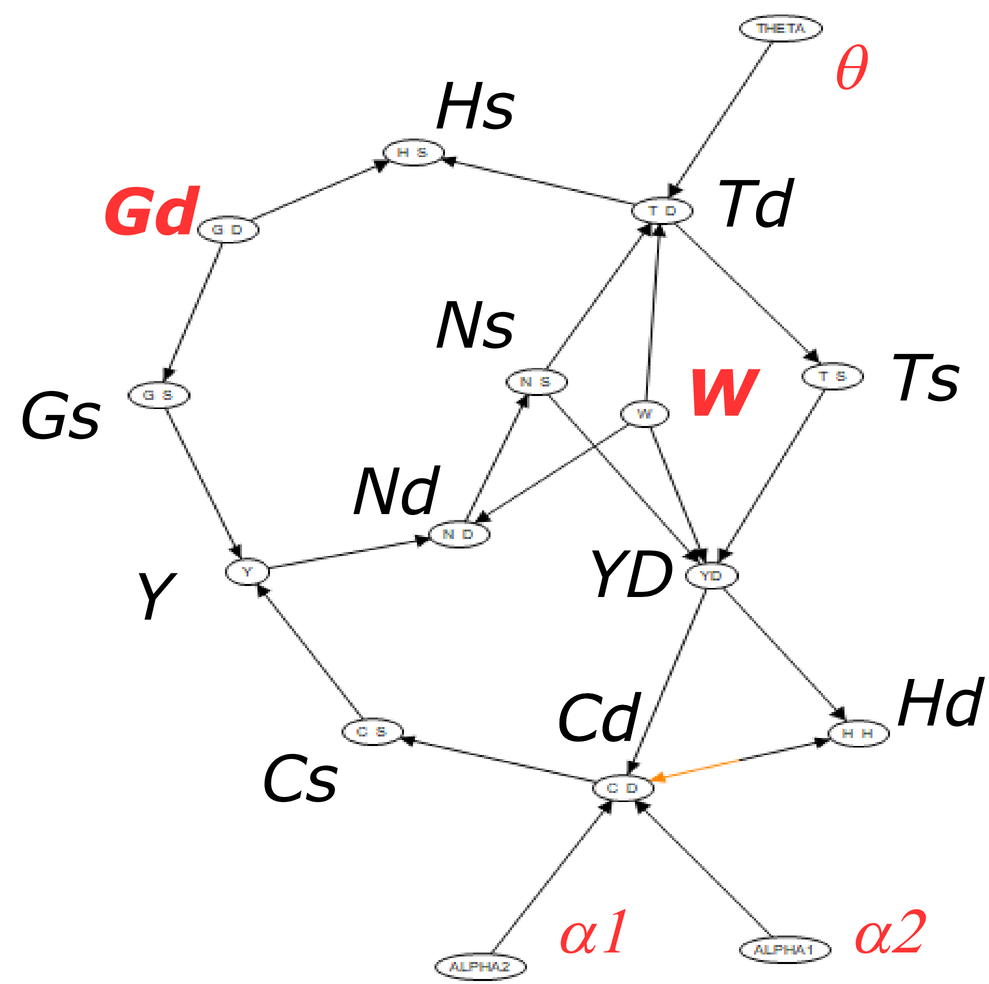
In our simple economy, the supply of financial asset is a result of independent decisions from the government, while the demand for financial asset is the outcome of the saving/consumption decisions of household.

However, accounting constraints require

Hd = Hs

Even without imposing this identity to hold

Model graph


THETA
θ
Hs
H S
Gd
G D
T D
Td
Ns
N S
W
W
T S
Ts
Gs
G S
Nd
N D
Y
Y
YD
YD
Cd
Hd
H H
C S
Cs
C D
α1
α2
ALPHA2
ALPHA1

Model solution – short run

We can drop the d, s subscript, now that we have shown the direction of causation.

By substitution we have

$$Y = \alpha1.(Y-\theta.Y) + \alpha2.H_{-1} + G$$

$$Y = m.(G + \alpha2.H_{-1})$$

where m is the standard Keynesian multiplier

$$m = 1/[1 - \alpha1.(1-\theta)]$$

Steady state solution

The model achieves a steady state solution when stocks stabilize. In our case, when

$\Delta H = 0$

Since

$\Delta H = G - T = G - \theta . Y$

in steady state we have

$Y = G/\theta$

Steady state solution #2

and

$\Delta H = YD - C$

Since

$C = YD = \alpha 1.YD + \alpha 2.H$

$(1 - \alpha 1).YD = \alpha 2.H$

$(1 - \alpha 1).(1-\theta).Y = \alpha 2.H$

$H/Y = (1 - \alpha 1).(1-\theta)/\alpha 2$

Choosing parameter values

Choosing parameter values is one of the challenges when developing theoretical SFC models. Analytical solutions help

Given our equations, choosing "reasonable" values for the tax rate and the propensities to consume:

$\theta = 0.2$; $\alpha 1 = 0.75$; $\alpha 2 = 0.05$

G = 100 implies

$Y = G/\theta = 100/0.2 = 500$

$H/Y = (1 - \alpha 1).(1-\theta)/\alpha 2 = 2000$

A numerical example

From the spreadsheet

Some notes on structural models

All (linear) dynamic models can be expressed in matrix form as

$$B.Y_t = G(L).Y_t + F(L).X_t \{+ u_t\}$$

which is the <u>structural form</u> of the model, and where *L* is the lag operator such that

$$L.Y_t = Y_{t-1}$$

The simplified model SIM

$$Y_t = C_t + G_t$$

$$C_t = \alpha 1.(1-\theta).Y_t + \alpha 2.H_{t-1}$$

$$H_t = H_{t-1} + G_t - \theta.Y_t$$

$$1.Y_t - 1.C_t = +1.G_t$$

$$\alpha 1.(1-\theta).Y_t + 1.C_t = + \alpha 2.H_{t-1}$$

$$- \theta.Y_t +1.H_t = 1.H_{t-1} + 1.G_t$$

The simplified model SIM

$$\begin{bmatrix} 1 & -\alpha_1 \cdot (1-\theta) & 0 \\ -1 & 1 & 0 \\ 0 & -\theta & -1 \end{bmatrix} \cdot \begin{bmatrix} C \\ Y \\ H \end{bmatrix} = \begin{bmatrix} \alpha_2 \\ 0 \\ 1 \end{bmatrix} \cdot \left[L \cdot H \right] + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cdot \left[G \right]$$

Model solution requires the simultaneous determination of *C* and *Y*. With complex models, simultaneous interaction may make the model difficult to solve.

If the model can be made recursive, such problems would disappear.

The simplified model SIM

Assume now that consumption depends on expected disposable income, and that expectations are adaptive:

$C_t = \alpha 1.(1\text{-}\theta).YE_t + \alpha 2.H_{t\text{-}1}$

$YE_t = Y_{t\text{-}1}$

The **B** matrix is now triangular, and the model is recursive

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -\theta & -1 \end{bmatrix} \cdot \begin{bmatrix} C \\ Y \\ H \end{bmatrix} = L \cdot \begin{bmatrix} 0 & \alpha_1 \cdot (1-\theta) & \alpha_2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C \\ Y \\ H \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cdot [G]$$

Some notes on structural models

Consider that, when the model includes a vector of stochastic disturbances, and parameters have to be estimated, simultaneity in the structural model makes equation-by-equation estimates biased.

The *reduced form* of the model can be estimated, but identification of the parameters in the structural model becomes a problem.

$$Y_t = B^{-1} \cdot G(L) \cdot Y_t - 1 + B^{-1} \cdot F(L) \cdot X_t \{ + B^{-1} \cdot u_t \}$$

Some notes on structural models

Another problem is how to determine if a variable can be treated as exogenous (in *X*) or should be determined by the model (in *Y*)

A common solution – not adopted as such in the PK-SFC world – is to treat all variables as potentially endogenous. The reduced form becomes a VAR

$$Y_t = B^{-1} \cdot G(L) \cdot Y_t - 1 \{ + B^{-1} \cdot u_t \}$$